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ABSTRACT

The purpose of this project was to produce an integrated program that effectively improves adult literacy, affects employment, enhances employability, and provides career adaptability. The goals directing these efforts were to design a series of modules for teaching reading, writing, and math, and to provide adults with basic employment skills. This tutorial is to improve the mathematics skills needed for vocational nursing. The four modules include a discussion of the properties of mathematics, fractions, decimals and percents, and ratios and proportions. Included in each module are examples and problem sets. The modules within this project are appropriate for selected vocational clients as well as non-college attending adults such as those participating in Job Training Partnership Act programs. (MVL)

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A New Dimension In Learning

Adult Literacy Project

Mathematics For Vocational Nursing Tutorial Modules

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1988

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Introduction

It has been reported that 33 percent of the adult population is functionally illiterate working at or below the fourth grade level. The need for a successful adult literacy program is well established and verifies the need for new approaches to adult literacy.

The purpose of this project was to produce an integrated program that effectively improves adult literacy, affects employment, enhances employability, and provides career adaptability. The goals directing our efforts were: to design a series of personal/social skills modules; to design a series of modules for teaching reading, writing, and math, and; to design a series of modules providing adults with basic employment skills.

A task force of specialists including community college faculty and counselors, governmental agency representatives, and Coordinating Board personnel worked together to identify client needs, skill competencies needed to meet those needs, and to identify strategies for integration. The integration of these activities centered around the development of material which reinforces and compliments skills taught in each of the three areas. For example, the material used to teach the skills of reading also reinforces skills needed for personal/social development such as assertiveness training. Correspondingly, material used to teach the basic employment module is consistent with the client's chances of learning the competencies.

The modules within this project are appropriate for selected vocational clients as well as non-college attending adults such as those participating in Job Training Partnership Act programs.

MATHEMATICS MODULES

MODULE ONE: PROPERTIES OF ARITHMETIC

Discussion of the addition, subtraction, multiplication, and division of real numbers. Presented are the properties of real numbers such as the cummutative property and the associative property.

MODULE TWO: FRACTIONS

Detailed discussion of how to add, subtract, multiply, and divide fractions.

MODULE THREE: DECIMAL AND PERCENTS

Discussion of how to convert decimals to fractions and decimals to percents. Also presented is the addition, subtraction and multiplication of decimal numbers.

MODULE FOUR: RATIOS AND PROPORTIONS

Discussion of ratios and proportions and how to solve proportions.

Module One
Properties of Arithmetic

Addition, subtraction, multiplication and division are operations used to manipulate numbers. These operations are designed to follow certain basic rules. The rules, also called properties of real numbers, must ALWAYS be kept in mind when performing arithmetic.

Arithmetic consists of two basic operations, addition and multiplication. Two other operations, subtraction and division, are defined in terms of addition and multiplication. Let us look at a few of the properties of real numbers.

Given any two real numbers a and b the folowing statements are true for addition and multiplication.

	PROPERTY	ADDITION	MULTIPLICATION
1.	COMMUTATIVE	a+b = b+a	ab = ba
	Example	5+2 = 2+5	5x2 = 2x5
2.	ASSOCIATIVE	(a+b) + c = a + (b+c)	a(bc) = (ab)c
	Example	(3+5) + 4 = 3 + (5+4)	4(3x5) = (4x3)5

(Notice that when parentheses are used with multiplication, the times sign, x, is not usually written.)

3.	IDENTITY	a + 0 = a	a x 1 = a

The ADDITIVE IDENTITY is zero.

The MULTIPLICATIVE IDENTITY is one.

The identity for an operation is that value which leaves the other number unchanged.

Ex. $2 + 0 = 2$ 0 is the identity for addition.
Ex. $5 \times 1 = 5$ 1 is the identity for multiplication.

4. INVERSE a and -a a and $\frac{1}{a}$

-a is the ADDITIVE INVERSE of a.

$\frac{1}{a}$ is the MULTIPLICATIVE INVERSE of a.

The inverse of a number is that value which when operated on the given number will result in the identity of the given operation. In other words, a number operated on by its inverse results in the identity of that operation.

EX. $2 + (-2) = 0$

Original number is 2.
Operation is addition.
Inverse of 2 is -2.
Result is the additive inverse, zero.

EX. $5 \times \frac{1}{5} = 1$

Original number is 5.
Operation is multiplication.
Inverse of 5 is one-fifth.
The result is the multiplicative identity, one.

5. DISTRIBUTIVE $a(b + c) = ab + ac$ $ab + ac = a(b+c)$

$(a + b)c = ac + bc$ $ac + bc = (a+b)c$

EX. $2(3 + 4) = (2x3) + (3x4)$

$= 6 + 12$

$= 18$

EX. Given that x is some unknown number:

$2(x + 3) = 2x + 6$

$(3 + x)4 = (3x4) + 4x$

EXAMPLES OF INVERSES

Additive inverses:

-3 is the inverse of 3 check: $-3 + 3 = 0$

5 is the inverse of -5 $5 + (-5) = 0$

It may be helpful to notice that additive inverses always have the same number involved, BUT they are of different signs. In other words, the additive inverse of a number is the given number times a negative one. It is precisely due to the above statements that the product of two negative numbers must be positive! Multiplying a negative one is by mathematical definition asking for the inverse of the given number.

EX. $-(-2) = -1 \times (-2)$

In words: the inverse of a negative two is equal to a positive two.

EX. Using the distributive property, any negative can be rewritten as a negative one times a positive number.

$-5 \times (-6) = -1 \times 5 \times (-6)$

Using the associative property, we can group the 5 and -6 together.

$$-1 \times 5 \times (-6) = -1 \times (5 \times -6)$$
$$= -1 \times (-30)$$

This last statement is asking for the inverse of a negative 30 which we know is equal to a positive 30.

It is sometimes difficult for students who have not had much exposure to mathematical terminology to grasp the meaning behind these statements and properties. Until the understanding becomes clear with practice, memorize how to determine additive and multiplicative inverses, memorize the identities, and know that the product of two negative numbers must always be positive.

Multiplicative inverse:

EX. The multiplicative inverse of 2 is $\frac{1}{2}$.

Check: $2 \times \frac{1}{2} = 1$

2 and $\frac{1}{2}$ are inverses of each other.

EX. The multiplicative inverse of -3 is $-\frac{1}{3}$.

Check: $-3 \times (\frac{-1}{3}) = 1$

A negative three times a negative one-third gives the multiplicative identity which is a positive one.

EX. The multiplicative inverse of $\frac{1}{4}$ is 4.

Check: $\frac{1}{4} \times 4 = 1$

PROBLEMS SET I.

1. What is the additive identity?
2. What is the multiplicative identity?
3. $12 + 0 =$ _____
4. $42 - 0 =$ _____
5. $5 \times 1 =$ _____
6. $1 \times (-6) =$ _____
7. What is the additive inverse of 13? _____
8. What is the multiplicative inverse of 13? _____
9. What is the additive inverse of -4? _____
10. What is the multiplicative inverse of -4? _____
11. What is the multiplicative inverse of $\frac{1}{1}$? _____

SUBTRACTION AND DIVISION

What about subtraction and division? What type of operations are they? The definitions of subtraction and division are as follows:

SUBTRACTION: $a - b = a + (-b)$

In words: a minus b is equal to a plus the inverse of b.

DIVISION: $a \div b = a \times \frac{1}{b} - \frac{a}{b}$ or a/b

In words: a divided by b is equal to a times the inverse of b.

In this fractional form, a/b, a is called the <u>numerator</u> and b is called the <u>denominator</u>. Module two has an indepth discussion of

fractions.

Subtraction is really addition of an inverse; division is really multiplication of an inverse.

SUBTRACTION:

EX. $2 - 3 = 2 + (-3) = -1$

EX. $3 - 1 = 3 + (-1) = 2$

EX. $-4 - 2 = -4 + (-2) = -6$

DIVISION:

EX. $2 \div 3 = 2 \times \frac{1}{3} = \frac{2}{3}$

EX. $4 \div (-5) = 4 \times \frac{1}{5} = \frac{-4}{5}$

EX. $25 \div \frac{1}{5} = 25 \times 5 = 125$

ORDERS OF PRIORITY FOR ARITHMETIC OPERATIONS

Notice how the equation (2x3) + (3x4) was solved in the first example for the distributive property. The parentheses make the order of operations clear. The multiplication is to be performed first and the two results, products, are to be added to each other. Parentheses are often omitted when the statements are short or to save space. The parentheses are not really necessary since arithmetic operations have priorities assigned to them with operations of higher priority being performed first. Multiplication and division are of equal priority. They are of a

higher priority than addition and subtraction. Addition and subtraction are also of equal priority; however, note that anything in parentheses or brackets should be simplified first.

EX. $4 - 6 \times (2 + 3) = 4 - 6 \times 5$ (parentheses simplified first)

$= 4 - 30$ (then multiplication performed)

$= -26$ (subtraction performed last)

EX. $2 \times 3 - 4 \times 7$ can be rewritten as $(2 \times 3) - (4 \times 7)$

$2 \times 3 - 4 \times 7 = 6 - 28$

$= -22$

EX. $2 - 3 \times 4$ can be written as $2 - (3 \times 4)$

$2 - 3 \times 4 = 2 - 12$

$= -10$

EX. $3 \times 4 \div 2 - 2$ can be written as $[(3 \times 4) \div 2] - 2$

$3 \times 4 \div 2 - 2 = 12 \div 2 - 2$ multiplication

$= 6 - 2$ division

$= 4$ subtraction

In the above example, the multiplication was performed before the division. When two operations of the same priority are present, the operations are taken in order from left to right as they appear in the statement. Here are some more examples:

EX. $3 + 4 - 2 \times 5 = 3 + 4 - 10$ multiplication

$= 7 - 10$ addition

$= -3$ subtraction

EX. $3 - (-4) + 6 \times 2 = 3 - (-4) + 12$ multiplication

$= 3 + 4 + 12$ $-(-4) = 4$

$= 7 + 12$ addition

$= 19$

Don't forget: -(-4) is asking for the inverse of a negative four which is a positive four. Or, think of it as a negative and a negative make a positive. So, subtraction of a negative number becomes addition of a positive number.

PROBLEMS SET II

1. $25 - 13 = 25 + _____ = 12$
2. $37 - 25 = _____$
3. $4 - (-3) = _____$
4. $-4 - 2 = _____$
5. $-14 - (-3) = _____$
6. $14 \div 2 = _____$
7. $24 \div \frac{1}{2} = 24 \times _____ = 48$
8. $8 \div \frac{1}{4} = 8 \times _____ = 32$
9. $2 + 3 \div 3 - 1 = _____$
10. $15 - 3 \times 4 \div 2 = _____$
11. $(5 \times 2 - 1) \div 3 = _____$
12. $(2 + 5) \times 2 - 4 \times 2 = _____$
13. $3 - (-4) \times 2 - 10 = _____$
14. $[14 - (-2)] \div 4 = _____$

TERMINOLOGY

Now that you have some understanding of the basic arithmetic operations, let us look at some of the terminology that is associated with these operations.

The result of the addition of numbers is called a sum; subtraction is often referred to as taking the difference of two numbers. When multiplying, each number is referred to as a factor, and the result of multiplication is a product. The result of division is referred to as the quotient. The numerator is also called the dividend and the denominator can be called the divisor.

EX. In the following statement 2, 3 and 4 are factors of the product 24: $2 \times 3 \times 4 = 24$

Notice that 24 is also the product of several other factors:

$$1 \times 24 = 24$$
$$2 \times 12 = 24$$
$$3 \times 8 = 24$$
$$4 \times 6 = 24$$
$$2 \times 3 \times 4 = 24$$

EX. The difference between 5 and 7 is equal to the following statement: $5 - 7$.

EX. The difference between 10 and 14 is written as $10 - 14$. Note that the second number is subtracted from the first number mentioned after the word between.

EX. The sum of 2 and 8 is written as $2 + 8$.

EX. The difference of some unknown number called x and 3 is written as $x - 3$.

OTHER HELPFUL PROPERTIES TO RECALL:

1. No number can be divided by zero.

EX. $5 \div 0 = 5 \times \frac{1}{0}$ = not possible

IMPORTANT NOTE: One divided by zero is not the same as zero divided by one.

$\frac{1}{0}$ = not possible $\frac{0}{1} = 0$

Zero may be in the numerator of a fraction, but it cannot be in the denominator.

2. Any number multiplied by zero is equal to zero.

EX. $-5 \times 0 = 0$
$1000 \times 0 = 0$

3. By definition of the multiplicative inverse, any number divided by itself is equal to one.

EX. $3 \div 3 = 1$
$105 \div 105 = 1$

ANSWER SETS

SET I.

1) 0 2) 1 3) 12 4) 42 5) 5 6) -6 7) -13

8) $\frac{1}{3}$ 9) 4 10) $-\frac{1}{4}$ 11) 2

SET II.

1) (-13) 2) 12

3) 7; $4 + 3 = 7$

4) -6; $-4 - (-2) = -6$

5) -11; $-14 + 3 = -11$

6) 7; $14 \times \frac{1}{2} = \frac{14}{2}$

7) $\frac{2}{1}$

8) 4 9) 2

10) 9; $15 - (3 \times 4) \div 2 = 15 - (12 \div 2) = 15 - 6 = 9$

11) 3; $(10 - 1) \div 3 = 9 \div 3 = 3$

12) 6; $(2 + 5) \times 2 - 4 \times 2 = (7 \times 2) - (4 \times 2)$

$= 14 - 8 = 6$

13) 1; $3 - (-4) \times 2 - 10 = 3 - (-8) - 10$

$= 3 + 8 - 10 = 1$

14) 4; $(14 + 2) \div 4 = 16 \div 4 = 4$

Module Two
Fractions

Fractions are another way to represent the division of two numbers. Recall that $3 \div 2$ represents 3 times the inverse of 2 which is written as follows:

$$3 \div 2 = 3 \times \frac{1}{2} = \frac{3}{2}$$

Thus, 3 divided by 2 can be written as the fraction $\frac{3}{2}$.

The top part of the fraction is called the <u>numerator</u> and bottom part is called the <u>denominator</u>. The horizontal line is used to separate the two values. Sometimes a slash, /, is used instead of the horizontal line. For example, 3/4 also represents 3 divided by 4.

There are several different types of fractions each possessing a different form. We will discuss each type of fraction individually.

1. A <u>proper fraction</u> is a fraction in which the numerator is less than the denominator. EX. $\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{8}$

2. An <u>improper fraction</u> is a fraction in which the numerator is greater than the denominator. EX. $\frac{5}{3}$, $\frac{7}{2}$, $\frac{23}{14}$

3. A <u>complex fraction</u> is a fraction in which the numerator

or denominator or both are also fractions.

EX. $\frac{\frac{1}{2}}{3}$, $\frac{\frac{3}{2}}{3}$, $\frac{\frac{1}{2}}{\frac{2}{3}}$

4. A mixed number is a numerical value that contains a whole number plus a fractional amount.

EX. $2\frac{1}{3}$, $3\frac{5}{7}$, $-2\frac{2}{5}$

$2\frac{1}{3} = 2 + \frac{1}{3}$ (plus sign not written)

Be careful with negative mixed number; they are a little different.

$-2\frac{1}{5} = -2 + (\frac{-1}{5})$

Think of $-2\frac{1}{5}$ as $(-1) \times 2\frac{1}{5}$ which by the distributive property would be as follows:

$-2\frac{1}{5} = (-1) \times 2\frac{1}{5}$

$= (-1) \times (2 + \frac{1}{5})$ < by definition of a mixed number

$= -2 + (\frac{-1}{5})$ < distributive property

GRAPHIC REPRESENTATION OF A FRACTION

A fraction represents a portion of a whole object. The numerator is referred to as the dividend which tells you how many parts of the whole you want to work with. The denominator represents the total number of parts or segments you have divided the whole

into.

EX. $\frac{1}{2}$ graphically: $\frac{1 \text{ part}}{2 \text{ total parts}}$

$\frac{1}{4}$ graphically: $\frac{1 \text{ part}}{4 \text{ total parts}}$

To add two fractions the same number of total parts must be involved. In other words, <u>the denominators must be the same</u>.

EX. + =

$\frac{1}{2}$ + $\frac{1}{2}$ = $\frac{2}{2}$ = 1 whole

When the denominators are the same, the numerators are added together to give the numerator of the solution.

EX. + =

$\frac{1}{4}$ + $\frac{2}{4}$ = $\frac{3}{4}$

Notice that just the numerators are added together. The denominators have not been changed.

EX. $\frac{1}{8} + \frac{4}{8} = \frac{5}{8}$

The formal definition for the addition of two fractions with alike denominators is as follows:

For all real numbers a, b, and c, where b does not equal zero,

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$$

Notice the difference in the following definition for the multiplication of fractions:

For all real numbers a, b, c, and d, where b and d are not equal to zero,

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

The numerators are multiplied together and their product becomes the numerator of the final fraction. And the denominators are multiplied together to give the denominator for the solution.

EX. $\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$

EX. $\frac{2}{3} \times \frac{5}{3} = \frac{2 \times 5}{3 \times 3} = \frac{10}{9}$

EX. $\frac{4}{3} \quad \frac{2}{5} \quad \frac{4 \times 2}{3 \times 5} \quad \frac{8}{15}$

Multiplication will be dealt with in detail later in this module.

PROBLEMS SET I

1. $\frac{2}{5} + \frac{1}{5} =$ _____

2. $\frac{1}{4} + \frac{3}{4} =$ _____

3. $\frac{2}{3} + \frac{1}{3} =$ _____

4. $\frac{3}{5} + \frac{1}{5} =$ _____

5. $\frac{4}{9} + \frac{1}{9} =$ _____

6. $\frac{5}{7} \times \frac{2}{5} =$ _____

7. $\frac{1}{5} \times \frac{2}{5} =$ _____

8. $\frac{2}{3} \times \frac{4}{5} =$ _____

9. $\frac{7}{8} \times \frac{1}{3} =$ _____

10. $\frac{1}{2} \times \frac{1}{2} =$ _____

ADDITION OF FRACTIONS WITH DIFFERENT DENOMINATORS

What happens when there is a different number of total parts involved in adding two fractions?

EX. +

$\frac{1}{4}$ + $\frac{1}{2}$

In order to add or subtract two fractions, the denominators must be equal. We must rewrite one or both of the fractions to make the denominators the same. To make this necessary change, the multiplicative identity must be used.

Recall from module one that any number divided by itself is equivalent to one and that any number multiplied by one remains unchanged in value. This allows us to multiply any fraction by another fraction equivalent to one without changing the value of the original fraction.

EX. $\frac{1}{3} \times \frac{4}{4} = \frac{1 \times 4}{3 \times 4} = \frac{4}{12}$

We are multiplying by $\frac{4}{4}$ which is equivalent to one. So by the multiplicative identity we have not changed the value of the fraction; we have just changed its form! One-third and four-twelfths represent the same number. Look at the following illustration:

$\frac{1}{2}$ = $\frac{2}{4}$

$\frac{1}{2} \times \frac{2}{2}$ = $\frac{2}{4}$

To get back to our original problem, how do we find the sum of 1/4 and 1/2? We need to have a common denominator for the two fractions, and we want this denominator to be the smallest number into which both original denominators can be divided evenly with no remainder. This smallest value is known as the least common denominator (LCD). It is also referred to as the lowest common denominator. A discussion of how to determine the LCD is presented a little later in this module. For now, concentrate on how to change the fractions into equivalent fractions with new denominators.

Our goal is to multiply the original fraction by some number which will give us a fraction equal in value to the original fraction but with the desired denominator. The number we need can be thought of as a missing factor. (Recall from module one: factor times factor = product.)

EX. $\frac{1}{4} + \frac{1}{2}$ LCD = 4 Both denominators will divide evenly into 4

The 1/4 already possesses the desired denominator - we need only to change (rewrite) 1/2 as a fraction with a denominator of 4.

$$\frac{1}{2} \times \frac{(\ )}{(\ )} = \frac{\ }{4}$$

missing factor LCD

1/2 times our missing factor must equal a fraction with a denominator of 4. In order for the two fractions to be equal, the missing factor must be equal to one. Thus, the numerator and the denominator of the missing factor must always be the same.

THINK: What number do I have to multiply 2 by in order to get the LCD which is 4? You need to multiply by 2.

$$\frac{1}{2} \times \frac{(\ )}{(2)} = \frac{\ }{4}$$

need 2 x 2 to get the LCD

$$\frac{1}{2} \times \frac{(2)}{(2)} = \frac{2}{4}$$

missing factor equal to one

Thus $\frac{1}{2} = \frac{2}{4}$ The two fractions are equal in value.

Now that 1/2 is rewritten as 2/4, we have two fractions of like denominators to be added:

So, $\frac{1}{4} + \frac{1}{2} = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}$

8

Let us try a few more problems.

EX. $\frac{1}{3} + \frac{1}{9}$

LCD = 9

$\frac{1}{3} \times \frac{(\ )}{(\ )} = \frac{\ }{9}$

$\frac{1}{3} \times \frac{(\ )}{(3)} = \frac{\ }{9}$

$\frac{1}{3} \times \frac{(3)}{(3)} = \frac{3}{9}$

So, $\frac{1}{3} + \frac{1}{9} = \frac{3}{9} + \frac{1}{9} = \frac{4}{9}$

EX. $\frac{2}{3} + \frac{3}{4}$

LCD = 12

In this case, both fractions need to be rewritten.

$\frac{2}{3} \times \frac{(\ )}{(\ )} = \frac{\ }{12}$ $\qquad$ $\frac{3}{4} \times \frac{(\ )}{(\ )} = \frac{\ }{12}$

$\frac{2}{3} \times \frac{(\ )}{(4)} = \frac{\ }{12}$ $\qquad$ $\frac{3}{4} \times \frac{(\ )}{(3)} = \frac{\ }{12}$

$\frac{2}{3} \times \frac{(4)}{(4)} = \frac{8}{12}$ $\qquad$ $\frac{3}{4} \times \frac{(3)}{(3)} = \frac{9}{12}$

$\frac{2}{3} = \frac{8}{12}$ $\qquad$ $\frac{3}{4} = \frac{9}{12}$

So, $\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$

EX. $\frac{2}{7} + \frac{1}{14} + \frac{5}{28}$

LCD = 28
The first two fractions need to be rewritten.

$\frac{2}{7} \times \frac{(\ )}{(\ )} = \frac{\ \ }{28}$ $\qquad$ $\frac{1}{14} \times \frac{(\ )}{(\ )} = \frac{\ \ }{28}$

$\frac{2}{7} \times \frac{(\ )}{(4)} = \frac{\ \ }{28}$ $\qquad$ $\frac{1}{14} \times \frac{(\ )}{(2)} = \frac{\ \ }{28}$

$\frac{2}{7} \times \frac{(4)}{(4)} = \frac{8}{28}$ $\qquad$ $\frac{1}{14} \times \frac{(2)}{(2)} = \frac{2}{28}$

$\frac{2}{7} = \frac{8}{28}$ $\qquad$ $\frac{1}{14} = \frac{2}{28}$

So, $\frac{2}{7} + \frac{1}{14} + \frac{5}{28} = \frac{8}{28} + \frac{2}{28} + \frac{5}{28} = \frac{15}{28}$

DETERMINATION OF THE LEAST COMMON DENOMINATOR

How do we find the LCD? The simplest method (but not always the quickest) is to look at the largest denominator of any of the fractions involved. Then take multiples of this number until you reach a value into which the other denominators will divide evenly.

EX. $\frac{2}{3} + \frac{3}{4} + \frac{3}{8}$

The largest denominator is 8. The 4 divides evenly into 8 but 3 does not. So look at multiples of 8. Start with 2 x 8, then 3 x 8, then 4 x 8, etc. until a product is reached in which both 3 and 4 will divide into evenly.

2 x 8 = 16 3 does not divide into 16 evenly.

3 x 8 = 24 Both 3, 4 and 8 will divide evenly into 24 so this is our LCD.

LCD = 24

Rewrite the fractions with the new denominator of 24:

$\frac{2}{3} \times \frac{(\ )}{(\ )} = \frac{\ }{24}$ $\quad \frac{3}{4} \times \frac{(\ )}{(\ )} = \frac{\ }{24}$ $\quad \frac{3}{8} \times \frac{(\ )}{(\ )} = \frac{\ }{24}$

$\frac{2}{3} \times \frac{(\ )}{(8)} = \frac{\ }{24}$ $\quad \frac{3}{4} \times \frac{(\ )}{(6)} = \frac{\ }{24}$ $\quad \frac{3}{8} \times \frac{(\ )}{(3)} = \frac{\ }{24}$

$\frac{2}{3} \times \frac{(8)}{(8)} = \frac{16}{24}$ $\quad \frac{3}{4} \times \frac{(6)}{(6)} = \frac{18}{24}$ $\quad \frac{3}{8} \times \frac{(3)}{(3)} = \frac{9}{24}$

So, $\frac{2}{3} + \frac{3}{4} + \frac{3}{8} = \frac{16}{24} + \frac{18}{24} + \frac{9}{24}$

$$= \frac{16 + 18 + 9}{24} = \frac{43}{24}$$

SUBTRACTION OF FRACTIONS

Recall that subtraction is really addition of an inverse. It is the addition of the negative of the given number. For example, 5 - 2 = 5 + (-2). Five minus two is equal to five plus the inverse of two which is a negative two. Let us apply this to fractions.

EX. $\frac{3}{2} - \frac{1}{2} = \frac{3}{2} + \frac{(-1)}{2} = \frac{3 + (-1)}{2} = \frac{2}{2} = 1$

EX. $\frac{4}{3} - \frac{2}{5}$ $\quad$ LCD = 15

$\frac{4}{3} \times \frac{(\ )}{(5)} = \frac{\ }{15}$ $\quad \frac{2}{5} \times \frac{(\ )}{(3)} = \frac{\ }{15}$

$\frac{4}{3} \times \frac{(5)}{(5)} = \frac{20}{15}$ $\quad \frac{2}{5} \times \frac{(3)}{(3)} = \frac{6}{15}$

So, $\frac{4}{3} - \frac{2}{5} = \frac{20}{15} - \frac{6}{15} = \frac{20}{15} + \frac{(-6)}{15}$

$$= \frac{20 + (-6)}{15}$$

$$= \frac{14}{15}$$

EX. $\frac{1}{4} - \frac{(-1)}{9}$ LCD: look at multiples of 9

$2 \times 9 = 18$

$3 \times 9 = 27$

$4 \times 9 = 36$ (This is the first value in which 4 and 9 can both divide evenly.)

LCD = 36

$\frac{1}{4} \times \frac{(\ )}{(9)} = \frac{\ \ }{36}$ $\qquad$ $\frac{-1}{9} \times \frac{(\ )}{(4)} = \frac{\ \ }{36}$

$\frac{1}{4} \times \frac{(9)}{(9)} = \frac{9}{36}$ $\qquad$ $\frac{-1}{9} \times \frac{(4)}{(4)} = \frac{-4}{36}$

So, $\frac{1}{4} - \frac{(-1)}{9} = \frac{9}{36} + \frac{(-1)(-4)}{36}$

$$= \frac{9 + 4}{36} = \frac{13}{36}$$

PROBLEMS SET II

1. $\frac{1}{2} + \frac{1}{7} = ____$

2. $\frac{1}{5} + \frac{1}{6} = ____$

3. $\frac{1}{3} + \frac{2}{4} + \frac{1}{5} = ____$

4. $\frac{1}{5} + \frac{2}{15} + \frac{3}{30} = ____$

5. $\frac{1}{10} - \frac{2}{5} = ____$

6. $\frac{3}{8} - \frac{5}{8} = ____$

7. $\frac{1}{2} + \frac{2}{3} - \frac{3}{5} = ____$

8. $\frac{5}{8} - \frac{2}{4} + \frac{3}{4} = ____$

9. $\frac{4}{7} - \frac{1}{3} + \frac{3}{5} = ____$

10. $\frac{3}{7} + \frac{4}{21} - \frac{1}{3} = ____$

REWRITING IMPROPER FRACTIONS

You may have noticed that the result of some of our examples were improper fractions. Sometimes it is beneficial to leave the fraction in this form, but often it is necessary to have the fraction in mixed number form, expressed as a whole number plus a fraction.

Recall that an improper fraction has a numerator greater than its denominator. For example, 8/3 is an improper fraction. To write 8/3 as a mixed number divide 8 by 3 to get the whole number for the mixed number. The remainder will be the numerator of the fractional part of the mixed number. The denominator will remain the same.

EX. Rewrite the following improper fraction as a mixed number.

$\frac{8}{3}$

$$3\overline{)\,8}\quad \begin{matrix} 2 & \text{whole number} \\ -6 & \\ \hline 2 & \text{remainder} \end{matrix}$$

So, $\frac{8}{3} = 2\frac{2}{3}$

EX. $\frac{13}{11}$

$$11\overline{)\,13}\quad \begin{matrix} 1 & \text{whole number} \\ -11 & \\ \hline 2 & \text{remainder} \end{matrix}$$

So, $\frac{13}{11} = 1\frac{2}{11}$

CONVERSION OF MIXED NUMBERS TO IMPROPER FRACTIONS

The first method of conversion is a longer method but explicitly shows you the arithmetic involved. Look over this method for an understanding of the conversion process. Method two is quicker to use as long as you are sure you understand the manipulations you are performing.

METHOD 1

EX. $2\frac{3}{5} = 2 + \frac{3}{5}$

$= \frac{2}{1} + \frac{3}{5}$ Any whole number can be written as a fraction since any number divided by 1 remains the same.

Find the LCD and then add the fractions.

LCD = 5 $\frac{2}{1} \times \frac{(5)}{(5)} = \frac{10}{5}$

$= \frac{10}{5} + \frac{3}{5}$

$= \frac{10 + 3}{5}$

$= \frac{13}{5}$

EX. $-3\frac{3}{5} = -3 + \frac{(-3)}{5}$

$= \frac{-3}{1} + \frac{(-3)}{5}$ LCD = 5 $\frac{-3}{1} \times \frac{(5)}{(5)} = \frac{-15}{5}$

$= \frac{-15 + (-3)}{5}$

$= \frac{-18}{5}$

METHOD 2

EX. $2\ \frac{3}{5}$

step 1: Multiply the whole number times the denominator.

$2 \times 5 = 10$

step 2: Add the value from step 1 to the numerator of the fraction from the mixed number. The result is the numerator for the improper fraction.

$10 + 3 = 13$

step 3: The denominator of the improper fraction is the same as the denominator of the fraction from the mixed number.

$\frac{13}{5} = 2\ \frac{3}{5}$

EX. $-3\ \frac{3}{5}$

$$-3\ \frac{3}{5} = (-1) \times 3\ \frac{3}{5}$$

$$= (-1) \times \frac{(3 \times 5) + 3}{5}$$

$$= (-1) \times \frac{15 + 3}{5}$$

$$= (-1) \times \frac{18}{5}$$

$$= \frac{-18}{5}$$

MORE SHORT CUT EXAMPLES:

EX. $5\ \frac{2}{3} = \frac{(5 \times 3) + 2}{3} = \frac{15 + 2}{3} = \frac{17}{3}$

EX. $7\ \frac{3}{4} = \frac{(7 \times 4) + 3}{4} = \frac{28 + 3}{4} = \frac{31}{4}$

ADDITION OF MIXED NUMBERS

Step 1: Convert the mixed numbers into proper fractions.

Step 2: Find the LCD and rewrite the fractions if the denominators of the improper fractions are not alike.

Step 3: Add the fractions and write the result as a mixed number.

EX. $1\ \frac{2}{5} + 2\ \frac{1}{5}$

step 1: $1\ \frac{2}{5} + 2\ \frac{1}{5} = \frac{(1 \times 5) + 2}{5} + \frac{(2 \times 5) + 1}{5}$

step 2: $= \frac{7}{5} + \frac{11}{5}$

step 3: $= \frac{7 + 11}{5} = \frac{18}{5} = 3\ \frac{3}{5}$

EX. $2\ \frac{1}{3} + 4\ \frac{2}{5}$

step 1: $2\ \frac{1}{3} + 4\ \frac{2}{5} = \frac{(2 \times 3) + 1}{3} + \frac{(4 \times 5) + 2}{5}$

step 2: $= \frac{7}{3} + \frac{22}{5}$ LCD = 15

$= \frac{35}{15} = \frac{66}{15}$

step 3: $= \frac{101}{15} = 6\ \frac{11}{15}$

16

EX. $3\frac{1}{2} - 2\frac{1}{5}$

step 1: $3\frac{1}{2} - 2\frac{1}{5} = \frac{(3 \times 2) + 1}{2} - \frac{(2 \times 5) + 1}{5}$

step 2 $= \frac{7}{2} - \frac{11}{5}$ LCD = 10

$= \frac{35}{10} + \frac{(-22)}{10}$

step 3: $= \frac{35 + (-22)}{10} = \frac{13}{10} = 1\frac{3}{10}$

EX. $3\frac{1}{3} - (-2\frac{1}{7})$

step 1: $3\frac{1}{3} - (-2\frac{1}{7}) = \frac{(3 \times 3) + 1}{3} + \frac{(2 \times 7) + 1}{7}$

A negative times a negative make a positive.

step 2: $= \frac{10}{3} + \frac{15}{7}$ LCD = 21

$= \frac{70}{21} + \frac{45}{21}$

step 3: $= \frac{115}{21} = 5\frac{10}{21}$

PROBLEM SET III

Rewrite the following improper fractions as mixed numbers:

1. $\frac{5}{2}$ 2. $\frac{9}{5}$ 3. $\frac{17}{5}$

Rewrite the following mixed numbers as improper fractions:

4. $3\frac{2}{5}$ 5. $2\frac{5}{9}$ 6. $-4\frac{2}{5}$

Perform the indicated operation on the following mixed numbers:

7. $4\frac{2}{5} + 3\frac{1}{4}$ 8. $2\frac{1}{3} - 3\frac{1}{5}$ 9. $5\frac{1}{2} + 2\frac{1}{3}$

MORE ON MULTIPLYING FRACTIONS

As already stated, when you multiply fractions you multiply the numerators together and you multiply the denominator together.

EX. $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$

EX. $\frac{1}{5} \times \frac{11}{2} = \frac{11}{10} = 1\frac{1}{10}$

EX. $\frac{2}{3} \times \frac{6}{4} = \frac{12}{12} = 1$

EX. $\frac{5}{2} \times \frac{3}{10} = \frac{15}{20}$

Notice in the last example that in the fraction 15/20 both the numerator and the denominator are multiples of 5. (i.e. 15 is a product of 5 and another factor; 20 is also the product of 5 and some other factor.) Thus the common factor of 5 can be factored out of the numerator and denominator (factoring is the distributive property in reverse). Notice how 15/20 can be rewritten:

$$\frac{15}{20} = \frac{5 \times 3}{5 \times 4}$$

$$= \frac{(5)(3)}{(5)(4)}$$

We can use the associative property to group the 5's together and to separate the fraction into two fractions

$$= \frac{(5)}{(5)} \times \frac{(3)}{(4)}$$

$= 1 \times \frac{3}{4}$ And from the inverse property we know that anything divided by itself is equal to one.

$= \frac{3}{4}$

3/4 is a fraction in <u>lowest terms</u> (simplest form) since the numerator and denominator have no factors other than one in common.

Reduce the following fractions to lowest terms:

$$\frac{9}{27} = \frac{9 \times 1}{9 \times 3} = \frac{(9)(1)}{(9)(3)} = 1 \times \frac{1}{3} = \frac{1}{3}$$

$$\frac{12}{18} = \frac{2 \times 6}{3 \times 6} = \frac{(2)(6)}{(3)(6)} = \frac{2}{3} \times 1 = \frac{2}{3}$$

You may have heard of this process referred to as cancellation; but be <u>careful</u> what you cancel. Cancellation is <u>really</u> finding factors which are equivalent to one! Finding and getting rid of factors which are equal to one can simplify multiplication of fractions. Notice carefully the properties which allow us to manipulate the fractions in the following example:

EX. $\frac{2}{3} \times \frac{9}{4} = \frac{2 \times 9}{3 \times 4}$ Definition of multiplication of fractions

$= \frac{2 \times 9}{4 \times 3}$ Commutative property of multiplication

$= \frac{(2)(9)}{(4)(3)}$ Associative property of multiplication

$= \frac{(2)(9)}{(4)(3)}$ Cancelling factors equal to one

^ factor of 3 in common

^ factor of 2 in common

$$= \frac{(1 \times 2)}{(2 \times 2)} \times \frac{(3 \times 3)}{(1 \times 3)}$$

multiplicative inverses apply

$$= \frac{1}{2} \times 1 \times \frac{3}{1} \times 1$$

$$= \frac{1 \times 1 \times 3 \times 1}{2 \times 1}$$ Definition of multiplication of fractions

$$= \frac{3}{2}$$

Shorthand for cancellation:

EX. $\frac{2}{15} \times \frac{10}{6} = \frac{2 \times 10}{6 \times 15}$

^ factor of 5 in common

^ factor of 2 in common

$$= \frac{\overset{1}{\cancel{2}} \times \cancel{10}^{2}}{\underset{3}{\cancel{6}} \times \cancel{15}_{3}}$$ 5 divides into 10 & 15; 2 divides into 2 & 6

$$= \frac{1}{3} \times \frac{2}{3}$$

$$= \frac{2}{9}$$

SHORT CUT METHOD:

Look for the largest number which will divide evenly into both the numerator and the denominator. Since this number is divided by itself, it is equal to one and can be left out.

EX. $\frac{3}{4} \times \frac{5}{9}$ (3 divides evenly into 3 and 9.)

$$\frac{\overset{1}{\cancel{3}}}{4} \times \frac{5}{\underset{3}{\cancel{9}}} = \frac{1}{4} \times \frac{5}{3} = \frac{5}{12}$$

EX. $\frac{2}{3} \times \frac{9}{8}$ (2 divided evenly into 2 and 8 and 3 divides evenly into 3 and 9.)

$$\frac{\cancel{2}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{9}^{3}}{\cancel{8}_{4}} = \frac{1}{1} \times \frac{3}{4} = \frac{3}{4}$$

It is not necessary to write out all the steps for each simplification. They are shown here so you can understand the process step by step. Without the <u>understanding</u> of the steps involved, you may find yourself cancelling factors which really are not equal to one. If this happens you have changed the fractions to non-equivalent fractions and your answer will be incorrect. ALWAYS be careful when cancelling factors! Notice the difference in the two statments below:

<u>CORRECT</u>

$$\frac{2+3}{6} = \frac{5}{6}$$

<u>INCORRECT</u>

$$\frac{2+3}{6} = \frac{2+\cancel{3}^{1}}{\cancel{6}_{2}} = \frac{2+1}{2} = \frac{3}{2}$$

Note: $\frac{2+3}{6}$ is actually $\frac{2}{6} + \frac{3}{6}$

Cancellation is done on <u>factors</u> which deal with products. Thus any addition or subtraction present in the numerator or denominator must be performed before you try to cancel.

EX. $\frac{2+4}{16} = \frac{6}{16} = \frac{\cancel{2} \times 3}{\cancel{2} \times 8} = \frac{3}{8}$

MULTIPLICATION OF FRACTIONS AND MIXED NUMBERS

Step 1: Convert the mixed numbers into improper fractions.

Step 2: Simplify (cancel) any multiples of one if present.

EX. $2\frac{1}{4} \times 3\frac{1}{2} = \frac{9}{4} \times \frac{7}{2}$

$= \frac{63}{8}$ no common factors

EX. $3\frac{1}{5} \times 3\frac{1}{8} = \frac{16}{5} \times \frac{25}{8}$

$= \frac{\cancel{16}^{2}}{\cancel{5}_{1}} \times \frac{\cancel{25}^{5}}{\cancel{8}_{1}}$

$= \frac{2}{1} \times \frac{5}{1}$

$= 10$

MULTIPLICATION OF FRACTIONS AND WHOLE NUMBERS

As stated previously, a whole number can always be written as a fraction by dividing the whole number by one.

For example, $4 = \frac{4}{1}$ and $10 = \frac{10}{1}$.

EX. $\frac{2}{3} \times 4 = \frac{2}{3} \times \frac{4}{1} = \frac{8}{3}$

EX. $\frac{3}{4} \times 8 = \frac{3}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{1} = \frac{6}{1} = 6$

DIVISION OF FRACTIONS

Recall from module one that division can be written as multiplication. For example, four divided by 2 can be written as four times the inverse of 2.

Numerically, this was written $4 \div 2 = 4 \times \frac{1}{2}$.

This process of conversion can be applied to fractions as well; first you must be able to find the inverse of a fraction. The easiest method of finding a fraction's inverse is to just invert the fraction (i.e. switch the numerator and the denominator). For example, the inverse of 2/3 is 3/2. Notice how this satisfies our definition of inverses:

$$\frac{2}{3} \times \frac{3}{2} = \frac{2 \times 3}{3 \times 2} = \frac{6}{6} = 1$$

$\wedge$ identity for multiplication

So to divide two fractions we need to invert the fraction which occurs immediately after the division sign.

EX. $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$

$\wedge$ inverse of 5/7

EX. $\frac{3}{5} \div \frac{2}{9} = \frac{3}{5} \times \frac{9}{2} = \frac{27}{10}$ or $2\ \frac{7}{10}$

EX. $\frac{3}{7} \div \frac{6}{21} = \frac{\cancel{3}^{1}}{\cancel{7}_{1}} \times \frac{\cancel{21}^{3}}{\cancel{6}_{2}} = \frac{1 \times 3}{1 \times 2} = \frac{3}{2}$

PROBLEM SET IV

Multiply the following fractions; simplify first where possible:

1. $\frac{2}{5} \times \frac{1}{4}$

2. $\frac{3}{4} \times \frac{16}{6}$

3. $\frac{5}{8} \times \frac{6}{15}$

4. $5 \times \frac{2}{3}$

5. $3 \times \frac{4}{6}$

Divide the following fractions; simplify the fraction where possible when multiplying:

6. $\frac{5}{2} \div \frac{3}{4}$

7. $\frac{1}{3} \div 4$

8. $\frac{2}{3} \div \frac{4}{6}$

9. $5 \div \frac{5}{7}$

SIMPLIFYING COMPLEX FRACTIONS

The same method of inversion and multiplication described in the previous section applies to complex fractions. This should seem appropriate since complex fractions are just another notation used for the division of two fractions.

EX. $\dfrac{\frac{1}{2}}{\frac{3}{4}} = \frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{1}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{3} = \frac{2}{3}$

EX. $\dfrac{\frac{2}{3}}{\frac{5}{7}} = \frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$

EX. $\dfrac{2}{\frac{1}{4}} = 2 \div \frac{1}{4} = \frac{2}{1} \times \frac{4}{1} = \frac{8}{1} = 8$

EX. $\dfrac{2}{10} = \frac{2}{5} \div 10 = \frac{2}{5} \times \frac{1}{10} = \frac{\cancel{2}^{1}}{5} \times \frac{1}{\cancel{10}_{5}} = \frac{1}{25}$

PROBLEM SET V

Simplify the following complex fractions:

1. $\dfrac{3}{\frac{1}{4}}$

2. $\dfrac{\frac{3}{2}}{\frac{1}{4}}$

3. $\dfrac{\frac{1}{5}}{6}$

4. $\dfrac{\frac{2}{3}}{2}$

5. $\dfrac{\frac{2}{5}}{\frac{4}{10}}$

6. $\dfrac{\frac{2}{5}}{7}$

7. $\dfrac{\frac{1}{5}}{\frac{10}{2}}$

8. $\dfrac{9}{\frac{1}{2}}$

ANSWER SETS

SET I.

1. $\frac{3}{5}$
2. $\frac{4}{4} = 1$
3. $\frac{3}{3} = 1$
4. $\frac{4}{5}$
5. $\frac{5}{9}$
6. $\frac{10}{35}$
7. $\frac{2}{25}$
8. $\frac{8}{15}$
9. $\frac{7}{24}$
10. $\frac{1}{4}$

SET II.

1. $\frac{9}{14}$; LCD = 14 $\quad \frac{1}{2} \times \frac{(7)}{(7)} = \frac{7}{14} \quad \frac{1}{7} \times \frac{(2)}{(2)} = \frac{2}{14}$

2. $\frac{11}{30}$; LCD = 30 $\quad \frac{1}{5} \times \frac{(6)}{(6)} = \frac{6}{30} \quad \frac{1}{6} \times \frac{(5)}{(5)} = \frac{5}{30}$

3. $\frac{62}{60} = 1\ \frac{2}{60}$ LCD = 60 This is a good example of how this method of finding the LCD may not be quick. Notice that 60 = 3 x 4 x 5.

$$\frac{1}{3} \times \frac{(20)}{(20)} = \frac{20}{60} \quad \frac{2}{4} \times \frac{(15)}{(15)} = \frac{30}{60} \quad \frac{1}{5} \times \frac{(12)}{(12)} = \frac{12}{60}$$

$$\frac{1}{3} + \frac{2}{4} + \frac{1}{5} = \frac{20}{60} + \frac{30}{60} + \frac{12}{60} = \frac{20 + 30 + 12}{60} = \frac{62}{60}$$

4. $\frac{13}{30}$; LCD = 30 $\quad \frac{1}{5} \times \frac{(6)}{(6)} = \frac{6}{30} \quad \frac{2}{15} \times \frac{(2)}{(2)} = \frac{4}{30}$

$$\frac{1}{5} + \frac{2}{15} + \frac{6}{30} = \frac{6}{30} + \frac{4}{30} + \frac{3}{30} = \frac{6 + 4 + 3}{30} = \frac{13}{30}$$

5. $\frac{-3}{10}$; LCD = 10

6. $\frac{-2}{8}$

7. $\frac{17}{30}$; LCD = 30

8. $\frac{6}{8}$; LCD = 8 $\quad \frac{5}{8} - \frac{2}{4} + \frac{3}{4} = \frac{5}{8} - \frac{4}{8} + \frac{6}{8} = \frac{5 - 4 + 6}{8} = \frac{7}{8}$

9. $\frac{63}{105} = 1\ \frac{73}{105}$

10. $\frac{6}{21}$

SET III.

1. $2\ \frac{1}{2}$

2. $1\ \frac{4}{5}$

3. $3\ \frac{2}{5}$

4. $\frac{17}{5}$

5. $\frac{23}{9}$

6. $\frac{-22}{5}$; $-4\ \frac{2}{5} = (-1)(4\ \frac{2}{5}) = (-1)\frac{(22)}{5}$

7. $\frac{153}{20} = 7\ \frac{13}{20}$

8. $\frac{-13}{15}$; $\frac{7}{3} - \frac{16}{5} = \frac{35 = 48}{15} = \frac{-13}{15}$

9. $\frac{47}{6} = 7\ \frac{5}{6}$

SET IV.

1. $\frac{1}{10}$; $\frac{\cancel{2}^{1}}{5} \times \frac{1}{\cancel{4}_{2}} = \frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$

2. 2 ; $\frac{3}{4} \times \frac{16}{6} = \frac{1}{1} \times \frac{4}{2} = 1 \times 2 = 2$

3. $\frac{1}{4}$; $\frac{5}{8} \times \frac{6}{15} = \frac{1}{4} \times \frac{3}{3} = \frac{1}{4} \times 1 = \frac{1}{4}$

4. $\frac{10}{3}$; $5 \times \frac{2}{3} = \frac{5}{1} \times \frac{2}{3} = \frac{10}{3}$

5. 2 ; $3 \times \frac{4}{6} = \frac{3}{1} \times \frac{4}{6} = \frac{4}{2} = 2$

6. $\frac{10}{3}$; $\frac{\cancel{5}}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{3} = \frac{10}{3}$

7. $\frac{1}{12}$

8. 1

9. 7

SET V.

1. 12; $3 \times \frac{4}{1} = 12$

2. 6 ; $\frac{3}{2} \times \frac{4}{1} = \frac{12}{2} = 6$

3. $\frac{1}{30}$; $\frac{1}{5} \times \frac{1}{6} = \frac{1}{30}$

4. $\frac{1}{3}$; $\frac{\overset{1}{2}}{3} \times \frac{1}{\underset{1}{2}} = \frac{1}{3}$

5. 1 ; $\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{5}}} \times \frac{\cancel{10}^{2}}{\cancel{4}_{2}} = \frac{2}{2} = 1$

6. $\frac{2}{35}$; $\frac{2}{5} \times \frac{1}{7} = \frac{2}{35}$

7. $\frac{1}{25}$; $\frac{1}{5} \times \frac{\cancel{2}^{1}}{\cancel{10}_{5}} = \frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$

8. 18 $9 \times \frac{2}{1} = 18$

Module Three
Decimals and Percents

CONVERSION OF DECIMALS TO FRACTIONS

Decimal numbers are another notation for representing a fraction. The numerator of the corresponding fraction is the number to the right of the decimal point. The denominator is always some multiple of ten.

EX. $0.25 = \frac{25}{100}$

The easiest method of remembering what multiple of ten to use for the denominator is to write a one and follow it by as many zeros as there are digits in the numerator.

EX. $0.215 = \frac{215}{1000}$ < 3 digits
< 3 zeroes after the 1

EX. $2.1257 = \frac{1257}{10000}$ < 4 digits
< 4 zeroes after the 1

EX. $0.25 = \frac{25}{100}$ reduced to simplest form = $\frac{1}{4}$

What should be done when zeroes are embedded within the decimal number?

EX. $0.\hat{0}215 = \frac{0215}{10000}$ < 4 digits
< 4 zeroes

The leading zero in the numerator must be counted as a digit in determining the number of zeroes to put in the denominator. Normally, we do not write leading zeroes as part of a number; so the leading zero is not needed in the numerator.

$= \frac{0215}{10000} = \frac{215}{10000}$

EX. $0.0015 = \frac{0015}{10000} = \frac{15}{10000}$

What should we do with trailing zeroes?

EX. $2.150 = 2\ \frac{150}{1000}$ < 3 digits
< 3 zeroes

Would we change the fraction if we left the end zero off?

$= 2\ \frac{150}{1000} = 2\ \frac{15}{100}$

The fraction would not be changed if we leave off a trailing zero in BOTH the numerator and the denominator. This is actually dividing the numerator and denominator by ten!

$= 2\ \frac{15}{100} = 2\ \frac{3}{20}$ in simplest form

ADDITION AND SUBTRACTION OF DECIMALS

Step 1: To add or subtract decimals vertically line the numbers up by their decimal points.

EX. $2.5 + 1.56 + 3.501$ =
$$\begin{array}{r} 2.5 \\ 1.56 \\ \underline{3.501} \end{array}$$

Step 2: Fill in any empty places at the END of the numbers with zeroes to help you visualize the addition or subtraction.

=
$$\begin{array}{r} 2.500 \\ 1.560 \\ \underline{3.501} \end{array}$$

Step 3: Add or subtract the decimals as you would whole numbers. Add the columns up right to left leaving the decimal place in the same position.

$$= \begin{array}{r} 2.500 \\ 1.560 \\ 3.501 \\ \underline{7.361} \end{array}$$

$5+5+3 = 13$

Place the end digit of the sum in the bottom row and carry over the other digit to the next column.

Any whole number can be written in decimal form. For example:
$4 = 4 + \frac{0}{10} = 4.0$

EX. $3.01 + 4 + 7.8197$

step 1:
$$\begin{array}{r} 3.01 \\ 4.0 \\ \underline{7.8197} \end{array}$$

step 2:
$$\begin{array}{r} 3.0100 \\ 4.0000 \\ \underline{7.8197} \end{array}$$

step 3:
$$\begin{array}{r} 3.0100 \\ 4.0000 \\ 7.8197 \\ \hline 14.8297 \end{array}$$

EX. $3.517 - 2.81 = \begin{array}{r} 3.517 \\ \underline{- 2.810} \end{array}$

$$= \begin{array}{r} 3.517 \\ \underline{- 2.810} \\ 0.707 \end{array}$$

EX. $3.142 - 1.579 = \begin{array}{r} 3.142 \\ \underline{- 1.579} \end{array}$

$$\begin{array}{r} 2\ 10\ 13\ 12 \\ \wedge\ \wedge\ \wedge\ \wedge \\ 3.\ \not{1}\ \not{4}\ \not{2} \\ \underline{- 1.\ 5\ \ 7\ \ 9} \\ 1.\ 5\ \ 6\ \ 3 \end{array}$$

< need to borrow one digit from the next column (The one digit is actually equal to ten.)

^ (12 - 9 = 3)
^ (13 - 4 = 6)
^ (10 - 5 = 5)
^ (do not need to borrow a digit; so left 3

with 2 - 1 = 0)

EX. 35.2714 - 27.8239 = 35.2714
-27.8239

2 14 12 6 10 14
^ ^ ^ ^ ^ ^
3 5 . 2 7 1 4
-2 7 . 8 2 3 9
7 . 4 4 7 5

^(14 - 9 = 5)
^(10 - 3 = 7)
^(6 - 2 = 4)
^(12 - 8 = 4)
^(14 - 7 = 7)
^(2 - 2 = 0)

MULTIPLICATION OF DECIMALS

To multiply two decimals line the numbers up vertically. Multiply as you would whole numbers. The position of the decimal point is equal to the number of digits to the right of the decimal in both numbers being multiplied.

EX.
```
   2.031   < 3 digits to the right of the decimal
x  3.15    < 2 digits to the right of the decimal
  10155
  20310
 609300
 ------
 639765

6.39765 < total of 5 places should be to the left
          of the decimal in the solution
```

EX.
```
  3.15   < 2 places to right of decimal
x 4.2    < 1 place to the right of decimal
  630
12600
-----
13230

13.230 < 3 places to left of decimal
```

^ If a zero occurs at the end of the number, it can be eliminated AFTER the decimal has been positioned.

13.230 = 13.23
solution = 13.23

CONVERSION OF FRACTIONS TO DECIMAL FORM

To convert fractions to decimal form perform the division indicated by the fraction. Recall that 1/4 is 1 divided by 4. By performing the long division, we have the fraction's decimal

equivalent.

$$4\overline{)1.00}\quad 0.25$$

8
20
20

$1/4 = 0.25$

We can check our results by rewriting the decimal in fractional form and reducing the fraction to lowest terms.

$$\frac{1}{4} = 0.25 = \frac{25}{100} = \frac{1}{4}$$

EX. $\frac{15}{100}$

$$100\overline{)15.00}\quad 0.15$$

10 0
5 00
5 00

EX. $\frac{3}{4}$

$$4\overline{)3.00}\quad 0.75$$

2 8
20
20

DIVISION OF DECIMALS BY WHOLE NUMBERS OR DECIMALS

Divide 8.9628 by 3.201.

dividend (^) divisor (^)

Step 1: Write the problem in long division form.

Step 2: Count the number of places to the right of the decimal in the divisor. Move the decimal point in both the divisor and the dividend to the right the same number of positions; add zeroes if necessary.

Step 3: Write the decimal point in the answer place just above where it is positioned in the dividend.

Step 4: Divide the numbers.

step 1. $3.201\overline{)8.9628}$

step 2. $3.201\overline{)8.9628}$

^ Move the decimal of the divisor 3 places to the right.

$3.201\overline{)8.9628}$

Move the decimal of the dividend the same number of places.

step 3. $3201\overline{)8962.8}$

step 4.
$$\begin{array}{r} 2.8 \\ 3201\overline{)8962.8} \\ \underline{6402} \\ 25608 \\ \underline{25608} \end{array}$$

EX. Divide 37 by 251.3 to the nearest thousandth.

step 1: $251.3\overline{)37}$

step 2: $251.3\overline{)37.00}$

step 3: $2513\overline{)370.0}$

step 4:
$$\begin{array}{r} 0.147 \\ 2513\overline{)370.00} \\ \underline{2513} \\ 11870 \\ \underline{10052} \\ 28180 \\ \underline{17691} \\ 10489 \end{array}$$

DECIMALS TO PERCENTS

Percent means to divide by 100. For example:

$$5\% = \frac{5}{100} \text{ or } 0.05$$

Thus, percents are just an extension of decimals.

EX. $100\% = \frac{100}{100} = 1$

EX. $50\% = \frac{50}{100} = 0.5 \quad < \quad 100\overline{)50.0}$

$$\begin{array}{r} 0.5 \\ 100\overline{)50.0} \\ \underline{50.0} \end{array}$$

The easiest way to perform the division by 100 is to realize that

dividing by 100 is equivalent to moving the decimal two places to the left!

EX. $2.5\% = \frac{2.5}{100} = 0\ 2.5 = 0.025$

EX. $35\% = \frac{35}{100} = 35.0 = 0.35$

To convert a fraction to a percent we need to find the decimal equivalent of the fraction and multiply by 100. To multiply by 100 we move the decimal two places to the <u>right</u> and add the percent sign, %.

EX. $\frac{1}{4} = ____\%$

$1/4 = 0.25 = 0.25 = 25\%$

EX. $\frac{2}{5} = ____\%$

$2/5 = 0.4 = 0.40 = 40\%$

$$5\overline{)\,2.0}\quad 0.4$$

EX. $\frac{3}{4} = ____\%$

$3/4 = 0.75 = 0.75 = 75\%$

$$4\overline{)\,3.00}\quad 0.75$$

PROBLEM SET I

Convert the following decimals to a fraction:

1) 0.125 2) 0.230 3) 1.718 4) 0.015

Perform the indicated operations:

5) 6.02 + 3.1 + 0.71

6) 5.156 - 2.158

7) 4.218 - 3.104 + 2.1

Multiply the following decimals:

8) 2.01 x 3.1

9) 7.82 x 0.341

10) 2.4 x 3.561

Convert the following fractions to decimal form:

11) 2/3 12) 5/7

Perform the following division:

13) 8.217 ÷ 2.1 14) 12 ÷ 0.418

Convert the following percents to decimal form:

15) 12% 16) 215%

Convert the following fractions to percents:

17) 2/5 = _____% 18) 2/3 = _____%

19) 7/9 = _____% 20) 1/5 = _____%

ANSWER SET

1. $\frac{1}{8}$; $0.125 = \frac{125}{1000}$ SIMPLEST FORM : 1/8

2. $\frac{23}{100}$; $0.230 = \frac{230}{1000} = \frac{23}{100}$

3. $1\ \frac{359}{500}$; $1.718 = 1 + \frac{718}{1000} = 1 + \frac{359}{500}$

4. $\frac{3}{200}$; $0.015 = \frac{015}{1000} = \frac{15}{1000} = \frac{3}{200}$

5. 9.83 ;
$$\begin{array}{r} 6.02 \\ 3.10 \\ \underline{0.71} \\ 9.83 \end{array}$$

6. 2.998 ;
$$\begin{array}{r} 5.156 \\ \underline{-2.158} \\ 2.998 \end{array}$$

7. 3.214 ;
$$\begin{array}{r} 4.218 \\ \underline{-3.104} \\ 1.114 \end{array} \qquad \begin{array}{r} 1.114 \\ \underline{+2.100} \\ 3.214 \end{array}$$

8. 6.231 ;
$$\begin{array}{r} 2.01 \\ \underline{\times\ 3.1} \\ 201 \\ \underline{6030} \\ 6231 \end{array} > 6.231$$

9. 2.66662 ;
$$\begin{array}{r} 7.82 \\ \underline{\times\ 0.341} \\ 782 \\ 31280 \\ \underline{234600} \\ 266662 \end{array} > 2.666662$$

10. 8.5464 ;
$$\begin{array}{r} 3.561 \\ \underline{\times\ \ 2.4} \\ 14244 \\ \underline{71220} \\ 85464 \end{array} > 8.5464$$

11. $0.6\overline{6}$; 2/3 (6's keep repeating)

12. 0.714 (rounded)

13. 3.913 (rounded)

14. 28.71 (rounded)

15. 0.12 ; $12\% = \frac{12}{100}$

16. 2.15

17. 40% ; $0.40 = 40\%$

18. $66.\overline{6}\%$

19. $77.\overline{7}\%$

20. 20%

Module Four
Ratios and Proportions

A ratio is a fractional expression which shows the relationship between two quantities. Ratios can be represented by several notations. The division sign may be a slash, /, ÷ , or :. The last symbol, :, is sometimes referred to as the dot notation.

EX. Given that 2 teaspoons of a liquid is vanilla extract and 5 teaspoons of the same liquid is chocolate extract, what is the ratio of chocolate to vanilla extract in the liquid?

$\frac{5}{2}$ or 5/2 or 5:2 represents the ratio of chocolate to vanilla

The ratio of vanilla to chocolate extract is:

$\frac{2}{5}$ or 2/5 or 2:5

EX. The ratio of 3 parts of sugar per 7 parts of water can be represented as follows:

$\frac{3}{7}$ or 3/7 or 3:7

EX. Two thirds of a mixture is acid and one sixth is water. What is the ratio of water to acid?

$\frac{\frac{1}{6}}{\frac{2}{3}}$ This is a complex fraction which needs to be put in simplest form (refer to module two).

$\frac{1}{6} \div \frac{2}{3} = \frac{1}{6} \times \frac{3}{2} = \frac{1}{4}$

So, the ratio of water to acid is 1/4 or 1:4.

A proportion is a statement that two ratios are equal. It represents the relationship between quantities. A proportion can be written in fractional form or dot form.

EX. $\frac{2}{3} = \frac{6}{9}$ or $2:3::6:9$
The :: replaces the equal sign.

For a proportion to be true, the ratios must be equivalent fractions.

In other words, the result (quotient) of each division must be equal.

$\frac{2}{3} = 0.66\overline{6}$ and $\frac{6}{9} = 0.66\overline{6}$

Comparisons of ratios can be used to solve for unknown quantities.

EX. If it takes 312 pounds of fertilizer to fertilize 3 lawns, how many pounds of fertilizer are needed to fertilize 2 lawns?

We are given one ratio: $\frac{312 \text{ pounds}}{3 \text{ lawns}}$ or 312.3

We want to know the numerator of the second ratio:

$\frac{X \text{ pounds}}{2 \text{ lawns}}$ or $X:2$

Using the two ratios, we set up the following proportion:

$\frac{312}{3} = \frac{X}{2}$ $312:3::X:2$

To solve the equation we have to apply some of our algebraic rules. We know that as long as we perform the same operation to <u>both</u> sides of the equals sign, the equation is still true. The easiest way to solve an equation involving fractions is to remove the fractions somehow! What happens if we multiply both sides of the equation by 3 and by 2?

$3 \times \frac{312}{3} = \frac{X}{2} \times 3$ Multiply both sides by 3

$312 = \frac{3X}{2}$ The 3's cancel on the left since 3/3 = 1.

$3 \times 312 = \frac{3X}{2} \times 2$ Multiply both sides by 2.

$624 = 3X$ The 2's cancel on the right since 2/2 = 1.

Now that the fractions are gone we need to get the X by itself. We want to know the value of X. Since the X is being multiplied by 3, if we divide both by 3 we can cancel the 3's and get X by itself.

$$\frac{624}{3} = \frac{3X}{3}$$ Divide both sides by 3.

$$208 = X$$ 3/3 = 1

Therefore, it takes 208 pounds of fertilizer to fertilize 2 lawns.

SHORT CUT: Instead of carrying out each multiplication and cancellation separately, we can cross multiply the two fractions. Cross multiplication is the net result of the longer method. Cross multiplication means to multiply the numerator of one ratio times the denominator of the other ratio.

EX. $\frac{312}{3} = \frac{X}{2}$

$\frac{312}{3} = \frac{X}{2}$

$2 \times 312 = 3 \times X$

$624 = 3X$ We can solve for X as previously shown.

$208 = X$

We can check our answers by comparing the two ratios. Their cross products must be equal.

$\frac{312}{3} = \frac{208}{2}$

$\frac{312}{3} = \frac{208}{2}$

$2 \times 312 = 3 \times 208$

$624 = 624$

EX. Two eggs are needed for half a batch of cookies. How many batches of cookies can be made with 12 eggs?

first ratio given: $\frac{2 \text{ eggs}}{\frac{1}{2} \text{ batch}} = 2 \div \frac{1}{2}$ or 2:0.5

$$2 \div \frac{1}{2} = 2 \times \frac{2}{1} = \frac{4}{1}$$

desired ratio: $\frac{12 \text{ eggs}}{X \text{ batches}} = 12 \div X$ or 12:X

proportion: $\frac{4 \text{ eggs}}{1 \text{ batch}} = \frac{12 \text{ eggs}}{X \text{ batches}}$

$$\frac{4}{1} = \frac{12}{X}$$

$$1 \times 12 = 4 \times X$$

$$12 = 4X$$

Divide both sides by 4 to get X by itself.

$$3 = X$$

Therefore, 3 batches of cookies can be made with 12 eggs.

Check: $\frac{4}{1} = \frac{12}{3}$

$$4 \times 3 = 1 \times 12$$
$$12 = 12$$

PROBLEM SET I.

1. $\frac{2}{3} = \frac{x}{12}$

2. $\frac{5}{11} = \frac{20}{x}$

3. $\frac{\frac{1}{2}}{3} = \frac{4}{X}$

4. $\frac{4}{0.25} = \frac{x}{1.75}$

5. 1:2::5:X

6. 3:4::X:16

7. 0.25:5::X:20

8. $\frac{1}{3}$: 3:: X: 9

ANSWER SET

1. 8 ; $2:3::X:12$ $\quad \frac{2}{3} = \frac{X}{12}$

$24 = 3X$
$8 = X$

2. 44 ; $5X = 220$

3. 24 ; $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6} = \frac{4}{X}$

$24 = X$

4. 28 ; $4 \times 1.75 = 0.25 \times X$

$7.0 = 0.25X$
$28 = X$

5. 10 ; $\frac{1}{2} = \frac{5}{X}$

6. 12 ; $\frac{3}{4} = \frac{X}{16}$

$48 = 4X$
$12 = X$

7. 1 ; $0.25 \times 20 = 5X$

8. 1 ; $\frac{\frac{1}{3}}{3} = \frac{X}{9}$

Can simplify complex fraction first:
$\frac{1}{9} = \frac{X}{9}$

Can cross multiply with the fraction:

$\frac{1}{3} \times 9 = 3X$

$3 = 3X$
$1 = X$